Public Key Encryption from the Worst-Case Shortest Vector Problem

Chris Peikert

SRI International

CRYPTO 2008 Rump Session

Lattice-Based Cryptography

What's To Like

- Simple & efficient: linear ops, small integers
- Resist subexp & quantum attacks (so far)
- Security from worst-case assumptions [Ajtai96,...]

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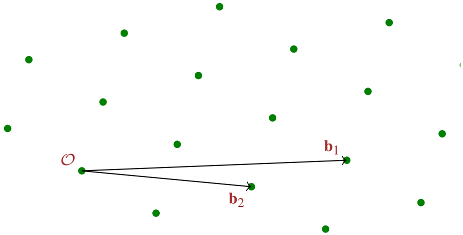

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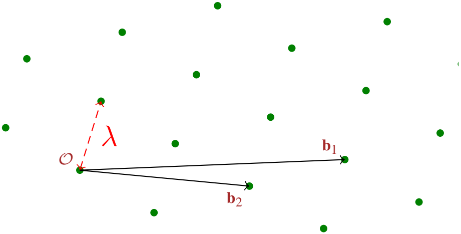

Shortest Vector Problem (GapSVP$_\gamma$)

- Given $\mathbf{B}$, is: $\lambda \leq 1$ or $\lambda > \gamma$?

Constructions and Assumptions

“Computational” Apps	
✔ CRHF	[Ajt96,PR06,LM06]
✔ ID schemes	[MV03,Lyu08]
✔ Signatures	[LM08,GPV08]

“Decisional” Apps	
✔ PKE	[AD97,Reg03,Reg05]
✔ CCA	[PW08]
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☞ GapSVP$_\gamma$ etc. hard

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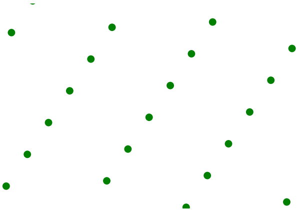
NOW: GapSVP$_\gamma$ hard

Prior Cryptosystems [AD97,Reg03,Reg05]

Main Reduction

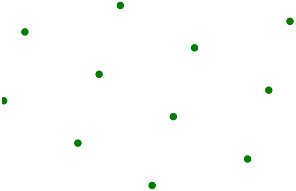
(small λ)



$\Downarrow$

Known “lumpy” dist

(large λ)



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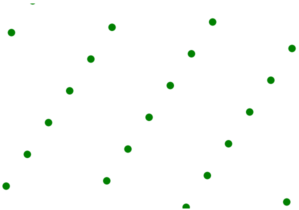

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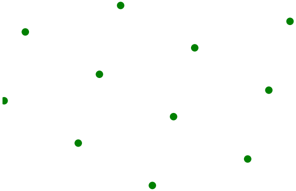
Normal public key

(large λ)



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Statistically-secure pk

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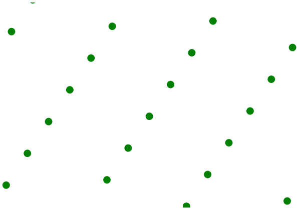

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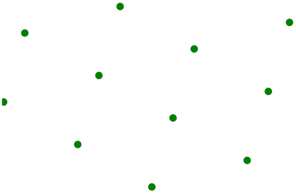

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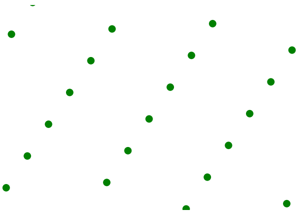
Statistically-secure pk

BOTH dists “fully specified” – need structured input.

Our Cryptosystem

Main Reduction



⇓

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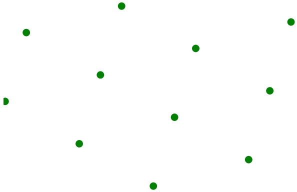

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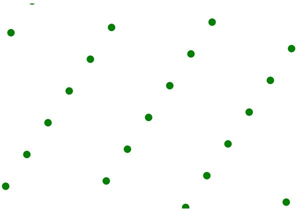

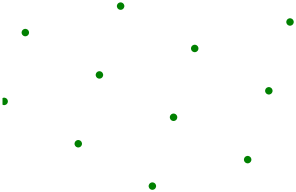

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$\mathcal{L}$ with small λ

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Uniform dist

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“Uniform” $\mathcal{L}$, **large** λ

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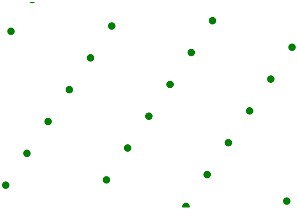

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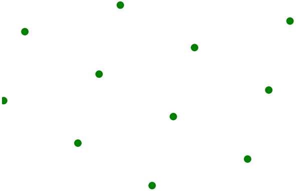

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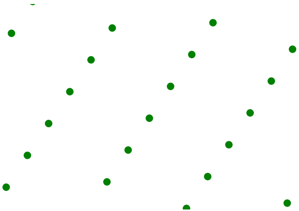

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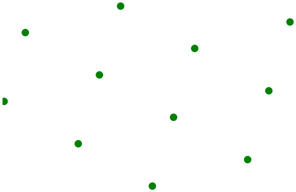

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Uniform dist

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"Uniform" $\mathcal{L}$, **large** λ

Normal public key

sk = "good" basis for $\mathcal{L}$

Miscellaneous

- Efficiency comparable to [AD97,Reg03] – potentially improvable
- 'Dequantize' [Reg05] ?

  Efficiency and apps: PKE, CCA, OT, IBE, . . .